



St Luke's School

Curriculum Progression Document

Maths



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### Curriculum Intent

### School Curriculum Intent:

As a values-led school, our curriculum is underpinned by Care, Challenge & Achieve. It is through these values that we develop the whole child. It is our intent that children leave St Luke's ready to move forward in their learning, kind, resilient, filled with a confidence to live well in society and prepared to deal effectively with the challenges that the modern world presents as well equipped digital and global citizens.

# Subject Intent:

It is our intent that children will learn through a mastery approach, which is deep, sustainable and achievable for all. They will have fluent knowledge and understanding of the number system with the ability to rapidly recall number facts, in addition to performing written and mental calculations efficiently. They will develop factual, conceptual and procedural fluency through a concrete, pictorial and abstract approach. Through a broad range of skills in applying mathematics, they will solve real life problems and reason about mathematical concepts and make connections. When faced with challenges in new and unusual contexts, children will think independently and persevere, showing confidence in success.

# Rationale for Decisions About What is Taught and When:

Supporting the NCETM's definition of mastery, the lesson sequence at St Luke's focuses on supporting children's understanding of core concepts and building their mathematical confidence. Each lesson is divided into evidence-based sections that take children on a journey through discovery, sharing of ideas, scaffolded practice, independent practice and reflection.

### Essential Characteristics in Maths:

- An understanding of the important concepts and an ability to make connections within mathematics.
- A broad range of skills in using and applying mathematics.
- Fluent knowledge and recall of number facts and the number system.
- . The ability to show initiative in solving problems in a wide range of contexts, including the new or unusual.
- The ability to think independently and to persevere when faced with challenges, showing a confidence of success.
- The ability to embrace the value of learning from mistakes.
- The ability to reason, generalise and make sense of solutions.
- Fluency in performing written and mental calculations and mathematical techniques.
- Able to use wide range of mathematical vocabulary, precisely and give answers in full sentences.
- A commitment to and passion for the subject.

# Key concepts our children will learn throughout their maths journey at St Luke:

Know and use numbers - understanding the number system and how they are used in a wide variety of mathematical ways.

Add and subject - understanding both the concepts and processes of addition and subtraction.

Multiply and divide - understanding both the concepts and processes of multiplication and division.

Use gractions - understanding the concept of part and whole and ways of calculating using it.

Understand the properties of shapes - recognising the names and properties of geometric shapes and angles.

Describe position, direction and movement - recognising various types of mathematical movements.

Use measures - becoming familiar with a range of measures, devices used for measuring and calculations.

Use statistics - interpreting, manipulating and presenting data in various ways.

Use algebra - recognising mathematical properties and relationships using symbolic representations.

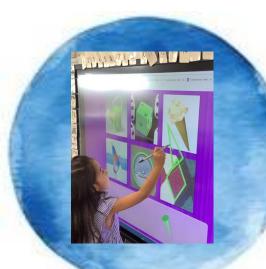
# Application/Problem Solving

At St Luke's, children solve mathematical problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions. In order to solve a problem, children will draw on one or more problem-solving skills, such as:

- Working systematically
- Trial and improvement
- Logical reasoning
- Spotting patterns
- Visualising
- Working backwards
- Conjecturing

# Through this way of teaching we will have:

- Ambilious expectations for all pupils
- · Gaps in learning immediately addressed through same day intervention
- All pupils access rich mathematical content
- Avoidance of grouping and labelling children
- Conceptual and procedural maths taught together
- Investment in professional development of teachers
- Children who know more and remember more









Maths in the Early Years Foundation Stage

Developing early Maths skills

### Developing Early Subject Skills — EYFS info The Unique Child reaches out to Prime areas are fundamental, relate to people and things through work together, and move through the Characteristics of Effective to support development in all other Learning, which move through all areas of learning. · Personal, Social and Emotional · playing and exploring Development . Communication and Language · active learning · creating and thinking critically · Physical Development Unique Child Specific areas include essential Children develop in the skills and knowledge for children to context of relationships participate successfully in society. and the environment around them. Literacy Mathematics This is unique to each family, and reflects . Understanding the World Understanding individual communities · Expressive Arts and Design and cultures.

Each area of the EYFS curriculum has an Early Learning Goal, which is the standard that a child is expected to achieve by the end of their reception year. The ELG (Early Learning Goals) covers all of the 7 areas of learning as specified in the Early Years Foundation Stage Curriculum.

The following link to the teaching and learning of Maths in our EYFS:

# ELG: Numbers, Shape, Space and Measures.

Children at the expected level of development will:

ELG II Numbers: Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

ELG 12 Shape, space and measures: Children use everyday language to talk about size, weight, capacity, position, distance, time and money to compare quantities and objects and to solve problems. They recognise, create and describe patterns. They explore characteristics of everyday objects and shapes and use mathematical language to describe them.



# Maths Specific Implementation/development of a programme of study

The programmes of study for mathematics are set out year-by-year for Key Stages I and 2. The starting point for teaching the programmes of study to a pupil is most likely to be the curriculum for the pupil's current year group. This may vary if pupils have identified Special Educational Needs, or have other significant barriers to learning e.g. non-attendance. Pupils would not usually be expected to progress to the curriculum above their year group. If these unusual circumstances are being considered, then advice should be sought from the Education Department.

# Key Stage 1 - Year 1 & Year 2

The principal focus of mathematics teaching in key stage I is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources for example, concrete objects and measuring tools. At this stage, pupils should develop their ability to recognise, describe, draw, compare and sort different shapes and use the related vocabulary. Teaching should also involve using a range of measures to describe and compare different quantities such as length, mass, capacity/volume, time and money. By the end of year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value. An emphasis on practice at this early stage will aid fluency. Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at key stage I.

# Lower Key Stage 2 - Year 3 & Year 4

The principal focus of mathematics learning in Lower Key Stage 2 is for pupils to develop their sense of number to increasingly larger whole number values and develop efficient and accurate methods to perform calculations on them. They are strengthening their appreciation of the numerical operations and place value. Calculators and other technology are used to support solving problems as well as develop conceptual understanding. Pupils should have begun to develop their concept of a number to include rational numbers (gractions expressed either as a quotient or as an extension to decimal notation). Pupils begin to apply mathematical operations to these numbers. Pupils should develop their ability to solve a range of problems, continuing to use concrete manipulatives and measuring tools, pictorial representations as well as numerals, and making connections between measure and number. Pupils should develop their mathematical reasoning so they can gain an awareness of geometrical structure, similar to seeing numerical structure. By the end of Year 4, pupils should be able to recall effectively multiplication facts for whole numbers less than or equal to 12. Pupils should be encouraged to draw with increasing accuracy and read and spell mathematical vocabulary correctly and confidently, using their growing reading knowledge and their knowledge of spelling. Teaching should always highlight the correct mathematical terminology to support this and insist that pupils communicate convincingly and unambiguously.

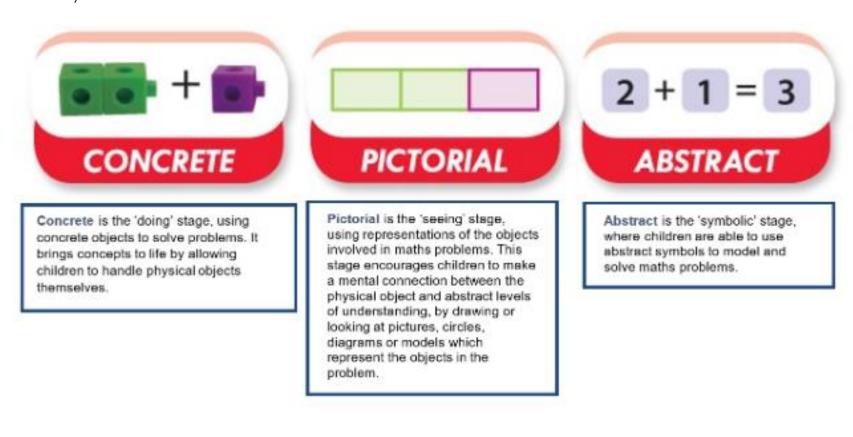
# Upper Key Stage 2 - Year 5 & Year 6

The principal focus of mathematics learning in Upper Key Stage 2 is for pupils to develop their sense of number to large integers, while using technology to significantly aid calculation. Pupils routinely verify results using approximation and an extended grasp of the place value system. Pupils develop connections between various numerical representations (different types of fractions, percentages and ratio) and continue to represent these with concrete and pictorial representations where appropriate. At this stage, pupils should develop their ability to reason with a wider range of problems, including increasingly complex properties of numbers. From this foundation in arithmetic (particularly through its pictorial representations), pupils

are introduced to the language of algebra as a means for solving a variety of problems. Learning to solve geometry problems with increasingly complex properties consolidates and extends their understanding in number. By the end of Year 6, pupils should be fluent in mental and written methods for all the four operations, when working with (at least two digit) integers, and be able to select the appropriate calculating methods, including the use of technology for a given question. They have an increasing fluency when working with rational (fractions, decimals percentages) and negative numbers and applying the four rules to them in both ID and 2D contexts. Pupils use technical vocabulary correctly to communicate their ideas and describe accurately the structure and solutions of their work.

At St Luke's children learn through using a high-quality textbook, online learning tools and a practice book for each term. There is a consistent use of the CPA (concrete, pictorial, abstract) approach, which helps children develop mastery across all the operations in an efficient and reliable way.

A calculations policy underpins the teaching of the 4 operations (addition, subtraction, multiplication and division) and clearly shows the concrete, pictorial and abstract approaches and progression in each strand of mathematics.



	<u>Aulumn</u>	<u>Spring</u>	<u>Summer</u>
Year I		Number Place value (within 20)	Multiplication and division (count in 2's, 10's and 5's)
	Number Addition and subtraction (within 10)	Number Addition and subtraction (within 20)	Number Fractions (half of quantities, groups and
	Geometry shape (2d and 3d shapes)	Number Place value (within 50)	shapes)
		Measurement Length and height (cm)	Geometry (position and direction, left/right and
		Measurement Mass and volume (heavy, lighter, mass)	forwards/backwards)
			Number Place value (within 100)
			Measurement (money)
			Measurement (time — months, hours minutes seconds,
			o'clock, half past)
Year 2	Number Place value (Numbers to 20, counting to 100,	Measurement Money (pence, pounds, make amounts,	Number Fractions (parts and whole, equal and
	making 10's, tens and ones, partition within 100)	compare money, make £1, find change and two step	unequal parks, recognise and find halves, quarkers and
	Number Addition and subtraction (bonds to 10 and 20,	problems)	thirds, unit and non unit fractions and equivalent
	add and subtract I's, add by making 10, add 3 one	Number Multiplication and Division (equal amounts/	fractions between a half and two quarters.
	digil no's, add / sublracl 2 digil no's across 10,	groups, x as a symbol, multiplication sentences, arrays,	Measurement Time (o'clock and half past, to and from
	compare numbers and missing number problems)	grouping/sharing, 2x tables, divide by 2, double and	the hour, tell the time @ 5 minute intervals, minutes in
	Geometry -Shape (2d and 3d shapes, symmetry, faces	halve, odd and even numbers)	an hour and hours in a day).
	and patterns).	Measurement Length and height (cm's, metres, length,	Statistics (tallies, tables block diagrams and
		height and four operations in height and length)	picłograms)
		Measurement Mass, Capacity and temperature	Geometry Position and Direction (movement, turns,
		(compare mass, grams, kilograms, volume, capacity,	patterns in turns and shape patterns with turns)
		millilitres, litres, four operations in volume and capacity	
		and temperature.	
Year 3	Number Place value (represent and partition numbers	Number Multiplication and Division B (multiples of 10,	Number Fractions B (add/subtract gractions, partition
	to 100, Hundreds, represent and partition numbers to	related calculations and reasoning about multiplication,	a whole, unit fractions of a set of objects, reasoning
	1,00, Hundreds, lens and ones, find 10 or 100 more or	2 digit by 1 digit multiplication with/without exchange,	with fractions of an amount,
	less, use number lines to 1,00)	link Multiplication and Division, 2 digit by I digit	Measurement Money (pounds pence and converting
	Number addition and subtraction (apply number bonds	Division with/without exchange, Division with flexible	pounds and pence, add/subtract money and find
	within 10, add/subtract 1's and 10's, 100's and spot the	partitioning and with remainders and scaling.	change).
	patterns. Add / subtract I's, across 10's, 10's across		

	Measurement Area	Number Decimals and percentages	Number Decimals	
	Number Multiplication and Division A	Measurement Perimeter and Area	Number Negative Numbers	
	Number Fractions A Statistics		Measurement Converting Units	
			Measurement Volume	
Year 6	Number Place value	Number Ratio	Geometry Shape	
	Number Addikion/Subkrackion	Number Algebra	Geometry Position and direction	
	Number Fractions A	Number Decimals	Themed projects, consolidation and problem solving.	
	<b>Number</b> Fractions B	Number Fractions, decimals and percentages.		
		Measurement Area, perimeter and volume		
		Statistics		







Whole School Maths Programme of Study



Curriculum Map — themes / topics			
	<u>Autumn</u>	<u>Spring</u>	<u>Summer</u>
Year I	Number: Place Value (within 10)	Number: Place Value (within 20)	Number: Multiplication and division
		Number. Fidde Value (within 20)	Number: Fractions
	Number: Addition and Subtraction (within 10)	Number: Addition and Subtraction (within 10)	Geometry: Position and direction
	Number: Naamon and Subtraction (within 10)		Number: Place Value (within 100)
		Number: Place Value (within 50)	Measurement: Money
	Geometry: Shape	Measurement: Length and height	Measurement: Time
	Consolidation	Measurement: Mass and volume	Consolidation
Year 2	Number: Place Value	Measurement: Money	Number: Fractions
	Number: Addition and Subtraction	Number: Multiplication and division	Measuremenl: Time
			Statistics
	Geometry: Shape	Measurement: Length and height	Geometry: Position and direction
		Measurement: Mass, capacity and temperature	Consolidation
Year 3	Number: Place Value	Number: Multiplication and division (B)	Number: Fractions (B)
		Measurement: Length and perimeter	Measuremenl: Money
	Number: Addition and Subtraction	Number: Fractions (A)	Measurement: Time
			Geometry: Shape
	Number: Multiplication and division (A)	Measurement: Mass and capacity	Statistics
			Consolidation
Year 4	Number: Place Value	Number: Multiplication and division (B)	Number: Decimals (B)
			Measurement: Money
	Number: Addition and Subtraction	Measurement: Length and perimeter	Measurement: Time

	<u>Autumn</u>	<u>Spring</u>	<u>Summer</u>	
	Measurement: Area		Consolidation	
	Number: Multiplication and division (A)	Number: Fractions	Geometry: Shape	
			Statistics	
	Consolidation	Number: Decimals (A)	Geometry: Position and direction	
Year 5	Number: Place Value	Number: Multiplication and division (B)	Geometry: Shape	
			Geometry: Position and direction	
	Number: Addition and Subtraction	Number: Fractions (B)	Number: Decimals	
	Number: Multiplication and division (A)	Number: Decimals and percentages	Number: Negative numbers	
	Number: Fractions (A)	Measurement: Perimeter and area	Measurement: Converting units	
		Statistics	Measurement: Volume	
Year 6	Number: Place Value	Number: Ratio	Geometry: Shape	
	Number: Addition, Subtraction, multiplication and	Number: Algebra		
	division	Number: Decimals	Geometry: Position and direction	
	Number: Fractions (A)	Number: Fractions, decimals and percentages	Themed projects, consolidation and problem solving	
	Number: Fractions (B)	Measurement: Area, perimeter and volume		
	Measurement: Converting units	Statistics		









### Approaches to Teaching and Learning in Maths.

Teaching and learning will focus on a range of agreed entitled experiences and there will be a focus on:

- Developing a clear progression of knowledge and skills linked to the essential learning objectives of the subject.
- Ensuring that appropriate opportunities are taken to develop cross-curricular skills such as English, Mathematics and Computing skills.
- The explicit teaching of how to work scientifically in each year group by carrying out a range of investigations and experiments.
- The consistent use of a range of teaching and learning approaches to engage pupils in the study of science. This will include objective and question led learning, observation and recording, class and group discussion, role play, handling a range of different materials, teaching of specific knowledge and retrieval practise activities.
- The study of important people, both male and female, who have influenced our understanding of science throughout history and in the modern world.
- The study of important scientific discoveries
- To use scientific knowledge to support, evaluate and challenge their own and others' views using detailed, appropriate and accurate scientific evidence derived from a range of sources. These are particularly relevant when understanding how the actions of others and their own actions impact on the world around them.
- The use of enrichment opportunities such as trips, visits and visitors.

### Maths Journals

Maths journals are an exercise book — grid, lined or plain paper — used by learners to record their methods, explanations and ideas for solving maths problems and learning of mathematical concepts. Maths journals provide teachers with insight into a learner's thinking and understanding of mathematical ideas, allowing them to assess and address misconceptions as they surface.

Learners should be encouraged to articulate and explain their thinking using written explanations, diagrams and numerical representations in their journals. Journals can be used almost as a diagnostic tool for both learners and teachers. When journalling learners may record what they already know about a topic, giving an indication of their starting point or for recording what the children have learned. It can often be interesting for learners and teachers to compare journals at the beginning of the unit with journals at the end of the unit.

# Journaling in EYFS/Yrl

- Floor books modelled explicitly by adults using leacher talk oracy tactics.
- Open ended questions to be used alongside modelled practical approaches.

# Journaling Y2

 Beginning the year recap and modelling of the process as taught in Year One, then moving onto children completing their own journal entries before October half term.

## Journaling in KS2

- Key vocabulary displayed during sessions for children to use.
- Sentences stems modelled and displayed.
- Manipulatives to be available
- Open ended questions used for journal type.

- Manipulatives to be available and modelled throughout.
- Evidence collated should include photographs and quotes from the children engaged in mathematical investigations.
- QR codes may also be used to record dialogue between adults and pupils or for recording individual summaries of the work completed.
- Journalling problems may be taken from White Rose for example: I wonder questions, help liny lime, flashback 4 etc.
- An adapted picture stimulus and question approach is needed for journaling starters to ensure the task reflects the needs of the learners in each cohort.

- Open ended questions to be used alongside modelled practical approaches.
- Manipulatives to be available and modelled.
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- Journalling problems may be taken from White Rose for example: I wonder questions, help tiny time, flashback 4 etc.
- An adapted picture stimulus and question approach is needed for journaling starters to ensure the task reflects the needs of the learners in each cohort.

- Problems/visuals taken from White Rose (adapted to the task as appropriate e.g. removal of extra text etc.)
- Pictorial representations
- Variety of learnt methods being showcased.

### Teaching, Recording, Feedback, Assessment and Reporting

# This will happen by:

- Learning Intentions are shared with children each lesson and displayed in children's books.
- Children are given a context through which they can explore each learning intention.
- The key knowledge for each unit is shared with children and parents through a knowledge organiser, which may include diagrams, key vocabulary, essential facts and key people associated with the learning. It will also highlight the key learning that will have taken place prior to this and pose questions which will form the basis of the learning enquiry.
- Teaching is focused on input, experiences and activities which promote the development of scientific understanding in the given area of learning at that time.
- The various methods of recording should demonstrate the children's understanding of the lesson's learning intention and how deeply they have understood the intention.
- Teachers' feedback should directly relate to the learning intention for the lesson, give specific ways in which the child has been successful.
- Gap Task and any verbal feedback where necessary.
- Children are given the opportunity to assess their own and others' progress. This may be recorded in books or done verbally.
- All Gap Tasks should be meaningful and purposeful and linked to small next steps for progress in science understanding and knowledge. They should be scaffolded where necessary.

- Teachers should use observations and work recorded by children to make judgements of the children's current progress against their year group's expectations.
- Teachers' judgements will also be informed by lessons outcomes and quizzes based on questions within the knowledge organisers.
- Regular retrieval practice focuses on children knowing and remembering more of what they have been taught previously.
- Assessment information will be used to plan future work for the class, including any intervention necessary.
- This continual assessment will be used to report to parents. End of year academic reports will contain comments about an individual pupil's progress against the year group expectations.
- All formative and summative assessments made will be used to inform discussions around pupils' progress and attainment in the subject at appropriate times, for example discussions with other professionals and reporting to parents on during parent consultation evening etc.
- Key scientist have been considered and chosen specifically for each area of focus and year group. Children will be introduced to these and links made to them in their learning. These scientists have been systematically planned over the year groups to allow progression in knowledge and understanding of the world around them.
- Children are exposed to, and will build, a range of age and topic related vocabulary from EYFS to year 6. This is found in the POS and builds on prior knowledge.
- Children are assessed at the end of each topic area. This is done through quizzes and assessment of understanding in lessons which is then collated by the class teacher and recorded on an assessment document. This can then be monitored by the science leader. Teachers can use this information for future planning opportunities / retrieval task / GT etc.

## Reading in Maths.

At St Luke's Primary School, reading is at the heart of the curriculum. It is our intent to ensure that every child not only develops the skills of reading but also a love of reading that will last them a lifetime. Our children read at home and school for pleasure, for information and to expand and enhance their knowledge and understanding across all subjects. Our children not only learn to read, they read to learn. Appropriate opportunities are taken to enhance children's learning in geography through reading with the use of high-quality texts across a wide range of genres. These are systematically matched to each topic in each year group, in order to impact on learning in the following ways:

- Knowledge of an extensive and subject-specific vocabulary.
- Fluency and accuracy in reading across a wide range of contexts throughout the curriculum.
- A desire to embrace challenging activities, including opportunities to undertake high-quality research, as well as interpret and present their findings.
- The ability to think, reflect, debate, discuss and evaluate processes.

The ability to consistently support, evaluate and challenge their own and others' views using detailed, appropriate and accurate mathematical knowledge derived from a range of sources.

### SMSC & Rights Respecting in Maths.

### What does this look like?

SMSC and Rights Respecting are not lessons which are taught in isolation, they are interwoven throughout our curriculum. Geography naturally provides rich opportunities for learning about the convention and there are clear links with global citizenship, sustainable development and similarities and differences across the world. Our staff have a deep understanding of

the United Nations Convention on the Rights of the Child (UNCRC) and are able to make links in lessons which are deep and meaningful. Staff are able to enhance teaching and learning by modelling rights respecting language and attitudes and making strategic decisions about the content of curriculum lessons that involve the children. Where appropriate, particular articles or areas of SMSC are linked to areas of geography to provide children with a broad knowledge and understanding.

# What impact does this have?

Due to the fact rights and SMSC development are integrated into our broad and balanced curriculum, children understand the importance of the convention and their SMSC key skills and it becomes a fundamental part of our school ethos. We have found that bringing a rights perspective to areas of the curriculum can enhance and enrich learning and instil a rights respecting ethos within our school. By ensuring that children have a rich SMSC and Rights Respecting understanding, we ensure that they are ready to embrace the challenges of creating a happy and successful adult life.

## Oracy - Progression of skills:

# Tiered Vocabulary Wall.

# A way to organise our words.

Tiered Vocabulary Walls are a way of organising words. The aim of using Tiered Vocabulary Walls is to increase the amount of Tier 2 and Tier 3 words which children hear and use themselves. Tier 2 and Tier 3 words make the most impact on our vocabulary and on our learning. These words need direct teaching in order for them to be understood and used.

Tier I - Everyday words: These will be basic, everyday words which will be used from an early age. These will be used freely in speech, such as:

warm, dog, lired, run, lable, flower...

<u>Tier 2 - Focus words:</u> These will be common words that are found across subjects. These will need direct teaching, such as: contradict, circumstance, precede, retrospect...

<u>Tier 3 - Subject specific words:</u> These will be rare and will be heard within particular contexts or subject areas. These will need direct teaching, such as: estuary, alliteration, igneous...

# Speaking like a Mathematician.

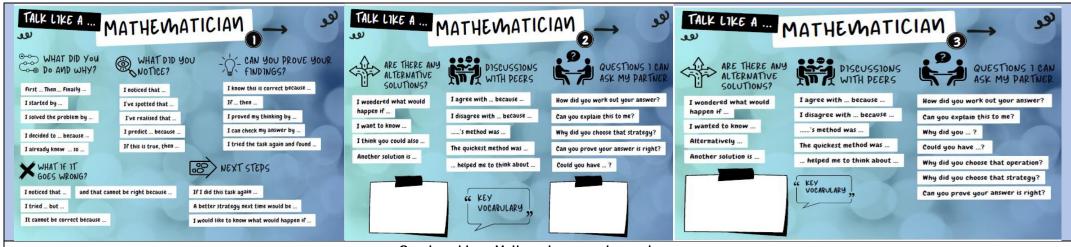
Speak concisely (keep it short!) so that you explain complex ideas in a way that is easy for others to understand.

Structure your ideas clearly, making sure that you have fully explained your scientific enquiry.

Use expert scientific vocabulary, but make sure that your audience understands it too.

Ask probing and clarifying questions to challenge others and developing your reasoning.

l.	2.	3.	4.	5.
Plans frequent exploratory talk	Uses manipulatives as a tool for	Connects classroom talk with	Teaches vocabulary explicitly,	Harnesses uncertainty to develop
opportunities	talk	being a mathematician.	according to a school-wide	deeper understanding
			progression.	



Speaking like a Mathematician sentence stems:

- It is the same / different...
- This reminds me of...
- I can prove I'm right because
- We must remember...because
- There is one more/one less...
- My working out is the same / different than yours because...
- I can prove I am right because...
- Another strategy you can use is...
- I learnt the word...and it means...
- We know that... so... it can't be...
- A major difference between... and... is that...
- I agree because...
- My strategy works because...
- I can check my answers by...
- Next time, I will...

- $\bullet\ \ I$  think the question means... so the answer means...
- I know that... Therefore, I would try out...
- I approached it methodically by...
- I was systematic... when...
- I looked at the whole problem and broke into these steps...
- So far, I have discovered that...
- The strategy I used was...
- I agree/disagree with...because...
- The solution makes sense because...
- I can visualise this problem by...
- I know my answer is reasonable because...
- $\bullet$   $\;$  The information needed to solve the problem is...
- When I used the inverse, I noticed...